EEEN 464 – DIGITAL COMMUNICATION

INTRODUCTION TO FOURIER SERIES - TEST YOUR KNOWLEDGE

MULTIPLE CHOICE QUESTIONS

1. What is the primary purpose of Fourier series in communication engineering?

a) To analyze time-domain signals
b) To represent periodic signals as a sum of sinusoids
c) To compress digital signals
d) To modulate carrier waves

Answer: b) To represent periodic signals as a sum of sinusoids

2. The Fourier series representation exists for:

a) All periodic signals
b) Only continuous signals
c) Signals satisfying Dirichlet conditions
d) Only analog signals

Answer: c) Signals satisfying Dirichlet conditions

3. The DC component in a Fourier series corresponds to:

a) The highest frequency component
b) The average value of the signal
c) The fundamental frequency
d) The phase shift

Answer: b) The average value of the signal

4. For an even periodic function, the Fourier series consists of:

a) Only sine terms
b) Only cosine terms
c) Both sine and cosine terms
d) Neither sine nor cosine terms

Answer: b) Only cosine terms

5. The Fourier series of a real-valued periodic signal has:

a) Complex coefficients
b) Only real coefficients
c) Symmetric coefficients
d) Both b and c

Answer: d) Both b and c

6. The fundamental frequency of a signal with period T is:

a) 1/T
b) T

c) 2π/T
d) T/2π

Answer: a) 1/T

7. In exponential Fourier series, the coefficients are:

a) Always real
b) Always imaginary
c) Generally complex
d) Always zero for odd harmonics

Answer: c) Generally complex

8. Parseval's theorem relates:

a) Time and frequency domains
b) Power in time domain to sum of squared coefficients
c) Phase and amplitude
d) Fundamental and harmonic frequencies

Answer: b) Power in time domain to sum of squared coefficients

9. The Gibbs phenomenon refers to:

a) Convergence at discontinuities
b) Overshoot near discontinuities
c) Loss of harmonics
d) Phase distortion

Answer: b) Overshoot near discontinuities

10. For a half-wave symmetric signal:

a) All even harmonics are zero
b) All odd harmonics are zero
c) DC component is maximum
d) Only cosine terms exist

Answer: a) All even harmonics are zero

11. The Fourier series coefficient a_0 represents:

a) Fundamental frequency amplitude
b) DC component
c) Phase shift
d) Highest harmonic

Answer: b) DC component

12. The trigonometric Fourier series represents a periodic signal as:

a) Sum of complex exponentials
b) Sum of sines and cosines
c) Product of harmonics
d) Integral of frequency components

Answer: b) Sum of sines and cosines

13. The Fourier series of a square wave contains:

a) Only odd harmonics
b) Only even harmonics

c) All harmonics
d) No harmonics

Answer: a) Only odd harmonics

14. The roll-off rate of Fourier coefficients indicates:

a) Signal bandwidth
b) Signal smoothness
c) Harmonic distortion
d) Phase linearity

Answer: b) Signal smoothness

15. In communication systems, Fourier series helps analyze:

a) Modulation schemes
b) Channel capacity
c) Bit error rate
d) Protocol efficiency

Answer: a) Modulation schemes

PROBLEM-SOLVING QUESTIONS

16. Find the Fourier series coefficients for $f(t) = 2 + 3\cos(100\pi t) + 4\sin(200\pi t)$

Solution:
This is already in Fourier series form:
$a_0 = 2$
$a_1 = 3$ (for 50 Hz)
$b_2 = 4$ (for 100 Hz)
All other coefficients = 0

17. Calculate the fundamental period of $x(t) = \cos(10t) + \sin(15t)$

Solution:
$\omega_1 = 10 \to T_1 = 2\pi/10 = \pi/5$
$\omega_2 = 15 \to T_2 = 2\pi/15$
Fundamental period $T = \text{LCM}(T_1,T_2) = 2\pi/5$

18. Find the exponential Fourier series coefficient c_n for f(t) = e^t, -π < t < π with period 2π

Solution:
c_n = (1/2π)∫[-π to π] e^t e^(-jnt) dt
= (1/2π)∫[-π to π] e^(t(1-jn)) dt
= (1/2π)[e^(π(1-jn)) - e^(-π(1-jn))]/(1-jn)

19. For a periodic square wave with amplitude A and period T, find the first three non-zero Fourier coefficients

Solution:
For odd square wave:
$a_n = 0$ for all n
$b_n = (4A)/(n\pi)$ for odd n, 0 for even n
First three non-zero terms:
$b_1 = 4A/\pi$
$b_3 = 4A/(3\pi)$
$b_5 = 4A/(5\pi)$

20. Using Parseval's theorem, calculate the power of $f(t) = 5 + 10\cos(50t + \pi/4)$

Solution:
$P = (5)^2 + (10/\sqrt{2})^2 = 25 + 50 = 75$

21. Determine if the signal $x(t) = t^2$, $-\pi < t < \pi$ has even or odd symmetry and what that implies for its Fourier series

Solution:
$x(-t) = (-t)^2 = t^2 = x(t)$ → Even function
Fourier series will have:

- Only cosine terms (all $b_n = 0$)
- Possible DC component

22. Find the Fourier transform of a periodic signal with known Fourier series coefficients c_n

Solution:
$F(\omega) = 2\pi \Sigma c_n \delta(\omega - n\omega_0)$
where ω_0 is fundamental frequency

23. Calculate the percentage of power contained in the first three harmonics of a square wave

Solution:
For square wave, power in nth harmonic $\propto 1/n^2$
Total power = $\Sigma (1/n^2)$ for odd n
First three harmonics: $1 + 1/9 + 1/25 \approx 1.1511$
Percentage = $(1.1511/1.2337)*100 \approx 93.3\%$
(where $1.2337 = \pi^2/8$ is total power)

24. Derive the relationship between trigonometric and exponential Fourier coefficients

Solution:
$c_n = (a_n - jb_n)/2$ for $n > 0$
$c_{-n} = (a_n + jb_n)/2$
$c_0 = a_0$

25. For $f(t) = |\sin(t)|$, determine if the function has half-wave symmetry

Solution:
$f(t + \pi) = |\sin(t + \pi)| = |-\sin(t)| = |\sin(t)| = f(t)$
Yes, it has half-wave symmetry (period π)

CONCEPTUAL QUESTIONS

26. Explain why Fourier series representation is important in communication systems

Answer:
Fourier series allows us to:

- Analyze signal frequency components
- Understand bandwidth requirements
- Design filters for specific frequency bands
- Study harmonic distortion
- Analyze modulation schemes

27. Describe the Gibbs phenomenon and its practical implications

Answer:
Gibbs phenomenon refers to the oscillatory overshoot (about 9%) near discontinuities when reconstructing signals with finite Fourier series. In practice, it:

- Limits perfect reconstruction of discontinuous signals
- Causes ringing artifacts in filtered digital signals
- Must be considered in filter design and signal processing

28. Compare and contrast Fourier series and Fourier transform

Answer:
Fourier series:

- For periodic signals
- Discrete frequency spectrum
- Represents signal as sum of harmonically related sinusoids

Fourier transform:

- For aperiodic signals
- Continuous frequency spectrum
- Generalization of Fourier series

29. Explain how Fourier series helps in analyzing harmonic distortion

Answer:
Fourier series decomposes distorted signals into:

- Fundamental frequency component
- Harmonic components (integer multiples)
- THD (Total Harmonic Distortion) can be calculated from coefficients
- Helps identify which harmonics are most problematic

30. Describe the convergence properties of Fourier series

Answer:

- Converges in mean-square sense for energy signals
- Pointwise convergence at points of continuity
- At discontinuities, converges to midpoint of jump
- Rate of convergence depends on signal smoothness
- Smoother signals have faster coefficient decay