SECTION A

Answer All 10 Questions

1. An Analog QAM system uses baseband messages $m_I(t)$ and $m_Q(t)$, each bandlimited to B Hz. What is the **transmission bandwidth** of the QAM signal?

◯ B Hz

◯ $2B$ Hz

◯ $B/2$ Hz

◯ $4B$ Hz

Explanation: Both the in-phase and quadrature components occupy the same carrier frequency, but the two sidebands overlap. Each message signal requires B Hz, and since they are transmitted simultaneously in quadrature, the total bandwidth is $2B$ Hz, from $f_c - B$ to $f_c + B$.

2. A 1.2 kHz sinusoidal signal is sampled at a rate of 1 kHz. What is the apparent frequency (in Hz) of the aliased signal after sampling?

◯ 200 Hz

◯ 800 Hz

◯ 1.2 kHz

◯ 2.2 kHz

Explanation: Aliasing causes frequencies above $f_s/2$ (500 Hz) to fold back. The folding frequency is $f_s/2 = 500$ Hz. The alias frequency is computed as $f_{alias} = | f_{signal} - kf_s |$, where k is an integer chosen so that $0 \leq f_{alias} \leq f_s/2$. For 1200 Hz: try $k = 1$: $| 1200 - 1000 | = 200$ Hz, which lies within 0–500 Hz. So apparent frequency = 200 Hz.

3. For a 16-point radix-2 FFT, how many stages of computation are required?

◯ 4

◯ 8

◯ 16

◯ 32

Explanation: For a radix-2 FFT with N = 2^m points, the number of stages is m. Since 16 = 2^4, m = 4 stages.

4. A bandpass signal has frequency components only between 12 kHz and 18 kHz. Its bandwidth is 6 kHz. What is the minimum sampling frequency allowed by the bandpass sampling theorem?

◯ 12 kHz

◯ 24 kHz

◯ 36 kHz

◯ 9 kHz

5. In a decimation-in-time FFT butterfly with inputs A and B, and twiddle factor W_k^N, what are the outputs? (Assume standard notation)

◯ $A + W_k^N \cdot B$ and $A - W_k^N \cdot B$

◯ A + B and A – B

◯ $W_k^N \cdot (A + B)$ and $W_k^N \cdot (A - B)$

◯ $A \cdot W_k^N + B$ and $A \cdot W_k^N - B$

Explanation: The basic butterfly computation for decimation-in-time FFT is:

Upper output = $A + W_k^N \cdot B$

Lower output = $A - W_k^N \cdot B$

6. A phase modulated signal has the form: $s(t) = A_c\cos(2\pi\times10^8 t + 20\sin(2\pi\times10^3 t))$. What is the modulation index?

◯ $2\pi\times10^3$

◯ 20

◯ $2\pi\times10^8$

◯ $2\pi\times10^{11}$

Explanation: The modulation index β for PM is the maximum phase deviation. In the expression s(t) = Accos(ωct + βsin(ωmt)), β is the coefficient of the sine term.

Here, β = 20, which is the maximum phase deviation in radians.

7. What is the bit-reversed order of index 6 (decimal) for an 8-point FFT? (Express in decimal)

◯ 3

◯ 6

◯ 4

◯ 5

Explanation: For N = 8 (3 bits):

6 in binary = 110

Bit-reversed = 011

011 in decimal = 3

8. For a real-valued input sequence, which symmetry property do the FFT outputs exhibit?

◯ X[k] = X[N-k] (periodic symmetry)

◯ X[k] = X*[N-k] (complex conjugate symmetry)

◯ X[k] = -X[N-k] (anti-symmetry)

◯ X[k] = X[k+N/2] (half-period symmetry)

Explanation: For real-valued x[n], the DFT exhibits complex conjugate symmetry: X[k] = X*[N-k] for k = 1, 2, ..., N-1. This means the magnitude is symmetric |X[k]| = |X[N-k]|, while the phase is antisymmetric.

9. How many complex multiplications are required for a 64-point radix-2 FFT?

◯ 64

◯ 192

◯ 384

◯ 4096

Explanation: For radix-2 FFT with N = 2^m:
Number of complex multiplications ≈ $(N/2)\cdot\log_2 N = (64/2)\cdot\log_2 64 = 32 \times 6 = 192$

10. You have a 32-sample sequence and zero-pad it to 64 samples before computing the FFT. What primarily changes in the resulting spectrum?

◯ The frequency resolution improves by factor of 2

◯ The spectral leakage decreases

◯ You get interpolated samples of the original 32-point DFT

◯ The Nyquist frequency increases

Explanation: Zero padding increases the number of frequency bins (interpolation in frequency domain) but does not improve true frequency resolution (which depends on original data length). It provides interpolated values of the original 32-point DFT.

SECTION II

Answer Any 3 Questions

11. Calculate the 4-point DFT of the sequence $x[n] = \{1,0,1,0\}$.

(Assume $n = 0,1,2,3$ *and* $X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$*)*

ANS

$N = 4, W_4 = e^{-j\frac{\pi}{2}} = -j.$
$X[k] = \sum_{n=0}^{3} x[n]\, W_4^{kn}.$

- $X[0] = 1 + 0 + 1 + 0 = 2$
- $X[1] = 1 + 0 \cdot (-j) + 1 \cdot (-j)^2 + 0 \cdot (-j)^3 = 1 + 1 \cdot (-1) = 0$
- $X[2] = 1 + 0 \cdot (-1) + 1 \cdot (1) + 0 \cdot (-1) = 2$

- $X[3] = 1 + 0 \cdot (j) + 1 \cdot (-1) + 0 \cdot (-j) = 0$

$$\boxed{X[k] = \{2,0,2,0\}}$$

12. A real-valued sequence $x[n]$ of length $N = 8$ has DFT $X[k]$. If $X[3] = 2 + 3j$, what is the value of $X[5]$?

ANS

For a real $x[n]$, DFT conjugate symmetry: $X[k] = X^*[N - k]$.
Here $N = 8$, so $X[5] = X^*[8 - 5] = X^*[3] = (2 + 3j)^* = 2 - 3j$.

$$\boxed{2 - 3j}$$

13. A signal is sampled at $f_s = 100$ Hz and 64 samples are collected. What is the frequency resolution (in Hz) of the resulting DFT?

ANS

Frequency resolution $\Delta f = \frac{f_s}{N} = \frac{100}{64} = 1.5625$ Hz.

$$\boxed{1.5625 \text{ Hz}}$$

14. The 4-point DFT of a sequence is $X[k] = \{4, 2j, 0, -2j\}$. Find the original sequence $x[n]$ using the inverse DFT formula.

ANS

Use IDFT: $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, W_N^{-kn}$, $N = 4$, $W_4 = -j$.

- $x[0] = \frac{1}{4}[4 + 2j + 0 + (-2j)] = \frac{1}{4}[4] = 1$
- $x[1] = \frac{1}{4}[4 + 2j \cdot (j) + 0 + (-2j) \cdot (-j)]$
  $$= \frac{1}{4}[4 + 2j \cdot j + 0 + (-2j) \cdot (-j)]$$
  $j \cdot j = -1$, so $2j \cdot j = -2$.
  $(-2j) \cdot (-j) = 2j \cdot j = -2$.
  So $x[1] = \frac{1}{4}[4 - 2 + 0 - 2] = 0$
- $x[2] = \frac{1}{4}[4 + 2j \cdot (-1) + 0 + (-2j) \cdot (-1)^3$? Wait carefully: $W_4^{-2} = (-j)^{-2} = (-1)^{-2} j^{-2} = 1 \cdot (-1) = -1$.
- So $X[1]W^{-2} = 2j \times (-1) = -2j$, $X[3]W^{-6} = (-2j) \times W^{-6} = (-2j) \times (W^{-2})^3 = (-2j) \times (-1)^3 = (-2j) \times (-1) = 2j$.
  Thus $x[2] = \frac{1}{4}[4 + (-2j) + 0 + 2j] = 1$
- $x[3] = \frac{1}{4}[4 + 2j \cdot (-j) + 0 + (-2j) \cdot (j)]$
  $2j \cdot (-j) = 2$, $(-2j) \cdot (j) = -2$.
  So $x[3] = \frac{1}{4}[4 + 2 + 0 - 2] = 1$.

Thus $x[n] = \{1,0,1,1\}$.

$\boxed{\{1, 0, 1, 1\}}$

15. A Pulse Amplitude Modulation (PAM) system samples a 4 kHz baseband signal at 12 kHz and transmits pulses with a duration of 20 μs. What is the null-to-null bandwidth of the transmitted PAM signal?

ANS

For a pulse duration $\tau = 20\ \mu s$, the **first null in the spectrum** is at $f = 1/\tau$.

$$1/\tau = 1/(20 \times 10^{-6}) = 50 \text{ kHz}$$

For PAM, the baseband spectrum repeats at multiples of $f_s = 12$ kHz, but the **null-to-null bandwidth of the main lobe** of each spectral copy is $2 \times (1/\tau) = 100$ kHz.
However, if we consider the **minimum transmission bandwidth** to preserve the pulse shape, it's often taken as $1/\tau$. For a rectangular pulse of width τ, the **null-to-null bandwidth =** $2/\tau$.
$B = 2/\tau = 2/(20 \times 10^{-6}) = 100$ kHz.